Joint list multistage decoding with sphere detection for polar coded SCMA systems

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***Abstract*—A reduced complexity sphere detection (SD) scheme for sparse code multiple access (SCMA) is presented, which employs an MMSE estimate of the transmitted vector to reduce the number of visited tree nodes. Simulation results show that the complexity of this method is lower compared to sphere decoding based on QR factorization, as well as message passing algorithm (MPA). Furthermore, joint list decoding algorithm for polar coded SCMA system is proposed, which provides better performance compared to the conventional sequential user partitioning method with independent list successive cancellation decoding.**

I. Introduction

Non-orthogonal multiple access (NOMA) enables one to significantly increase the number of supported users in wireless systems. In NOMA systems the number of orthogonal resource elements (REs) utilised by each user is much smaller than the total number of REs. Sparse code multiple access (SCMA) is a recently proposed instance of non-orthogonal multiple access [1]. It provides simple codebook construction techniques, as well as very good performance.

In [2] a message passing algorithm (MPA) is proposed for detection of SCMA symbols. However, the computational complexity of this method is quite high. An extension of the sphere decoding algorithm [3] to the case of SCMA was presented in [4]. Soft-output detection method based on sphere decoding was presented in [5].

In this paper we present a reduced complexity soft-output detection method for SCMA based on sphere decoding. The complexity reduction is achieved by employing a minimum mean square error (MMSE) estimate for the transmitted vector, which enables the sphere decoder to quickly find the optimal solution.

Furthermore, we observe that the sequential user partitioning (SUP) scheme, commonly used for SCMA detection, is very similar to the successive cancellation (SC) decoding algorithm for polar codes. Indeed, the estimates obtained at each step of these algorithms are used in the following steps. A standard way to improve the performance of polar codes is to employ list SC decoding [6]. We propose to extend the idea of list SC decoding to the case of polar codes concatenated with inner SCMA codes, and introduce a joint list decoding algorithm for polar coded SCMA. Simulation results show that it provides significant performance gain with respect to SUP with list SC decoding for each user.

The paper is organized as follows. Section II presents a brief survey of SCMA, sphere decoding and polar codes. The proposed improved sphere decoding algorithm is introduced in Section III. Section IV presents an extension of the proposed approach to the case of coded SCMA systems, as well as the joint list SC decoding algorithm. Simulation results are provided in Section V.

II. Background

A. System Model

Consider an SCMA system with J users and F orthogonal resource elements, $F < J$. Each user is assigned with an SCMA codebook $C_j, j = 0, \ldots, J-1$, which consists of M_j complex-valued vectors (SCMA code sequences) of length F, so that each vector contains $d_j \ll F$ non-zero elements.

Fig. 1 illustrates the transmitters in the SCMA system. For user j the channel encoder transforms the data vector v_j of size k to obtain codeword c_j of size n. The codeword is partitioned into T tuples of length $d' = \log_2 M_j$, $n = d'T$ and each tuple is replaced with the corresponding vector $x_{j,i}$ of size F, $i = 0, ..., T-1$, from the SCMA codebook C_j. We assume for the sake of simplicity that all users have the same $M_j = M$ and $d_j = d$.

An SCMA encoder performs a mapping of d' bits to the F-dimensional complex codebook C_j of size M. A mother constellation for SCMA [7] can be represented by matrix $\mathbf{M}_c = \mathbf{G}\mathbf{U}$, where $\mathbf{G}$ is a complex-valued generating matrix of size $d \times d'$, and $\mathbf{U}$ is a matrix of size $d' \times M$. For example, for codebooks shown in [8] the matrixes $\mathbf{G}$ and $\mathbf{U}$ can be defined as

$$\mathbf{G} = \begin{bmatrix} 0.8858 & 0.4640 \\ -0.2179 - 0.4097i & 0.4158 + 0.7821i \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}.$$

We assume that data bits are mapped onto elements of $\mathcal{Q} = \{-1, 1\}$, and the columns of matrix $\mathbf{U}$ correspond to all possible combinations of d' bits. Let $F \times d$ matrix $\mathbf{V}_j$ define a mapping of constellation points with d non-zero dimensions to the SCMA code sequences of codebook C_j. Furthermore, let $\mathbf{\Delta}_j$ be a $d \times d$ matrix, called the constellation operator, which represents phase rotation and permutations for user j. This operator is introduced to avoid interfering of the same dimensions of a mother constellation over a resource node [2]. Thus, a codebook C_j can be presented as $C_j = \mathbf{V}_j \mathbf{\Delta}_j \mathbf{M}_c$.

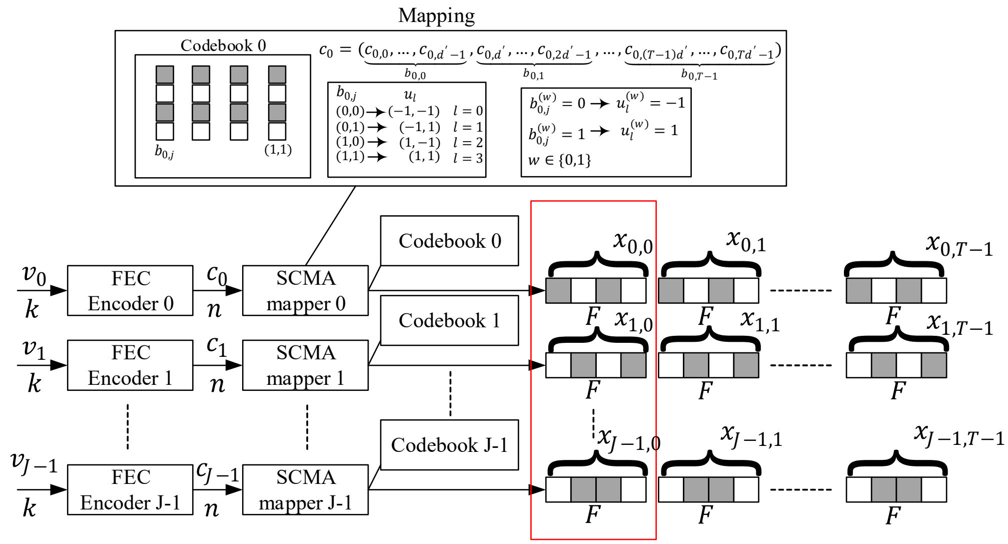

Fig. 1. SCMA transmitters with $M = 4$, $F = 4$, $d = 2$

The transmitted F-dimensional complex SCMA code sequence of the j-th user is given by

$$x_j = \mathbf{V}_j \mathbf{\Delta}_j \mathbf{G} u_j, \tag{1}$$

where u_j is the j-th column of matrix $\mathbf{U}$. For simplicity we consider only one block of transmitted SCMA code sequences, thus, we use notation x_j instead of $x_{j,i}$, $i = 0, ..., T-1$.

The receiver observes

$$y = \sum_{j=0}^{J-1} \operatorname{diag}(h_j) x_j + \eta, \tag{2}$$

where $y = (y_0, ..., y_{F-1})^T$, $h_j = (h_{j,0}, h_{j,1} ..., h_{j,F-1})^T$ are the channel gain coefficients, $x_j = (x_{j,0}, ..., x_{j,F-1})^T$ is a SCMA code sequence of user j, and η is a vector containing complex additive white Gaussian noise (AWGN) samples with zero mean and a covariance matrix $\sigma^2 I$.

Equation (1) implies that the received vector in (2) may be represented as

$$y = \sum_{j=0}^{J-1} \operatorname{diag}(h_j) \mathbf{V}_j \mathbf{\Delta}_j \mathbf{G} u_j + \eta = \mathbf{B} \underbrace{(u_0^T, \ldots, u_{J-1}^T)^T}_{u} + \eta,$$

where $\mathbf{B} = [\mathbf{B}_0, \mathbf{B}_1, ..., \mathbf{B}_{J-1}]$ is matrix of size $F \times Jd$, $\mathbf{B}_j = \operatorname{diag}(h_j) \mathbf{V}_j \mathbf{\Delta}_j \mathbf{G}$. Therefore, we consider the vectors u_j as a payload data.

B. Sphere Decoding

The detection problem for SCMA can be formulated as finding

$$\hat{u} = \arg\min_{u \in \mathcal{X}} ||y - \mathbf{B}u||^2, \tag{3}$$

where $\mathcal{X} = \mathcal{Q}^t$, $t = Jd$.

The main idea of sphere decoding is to search over those vectors that are located in a certain sphere of radius r around the given received vector y, so that

$$r^2 \geq ||y - \mathbf{B}u||^2. \tag{4}$$

The sphere decoding algorithm reduces to search over a tree, where the branches in the l-th level of the tree correspond to the vector of symbols inside a sphere of radius r and dimension l. The tree have t layers and each path corresponds to one of the possible values of vector u. Each not-leaf node has two output branches, which correspond to values $u_i \in \mathcal{Q}$.

Since u is a real-valued vector, it is convenient to work with a real-valued model. Let

$$\tilde{y} = \begin{pmatrix} \mathcal{R}(y) \\ \mathcal{I}(y) \end{pmatrix} \in \mathbb{R}^{2F}, \tilde{\mathbf{B}} = \begin{pmatrix} \mathcal{R}(\mathbf{B}) \\ \mathcal{I}(\mathbf{B}) \end{pmatrix} \in \mathbb{R}^{2F \times t}. \tag{5}$$

Then an equivalent real-valued model is given by

$$\hat{u} = \arg\min_{u \in \mathcal{X}} ||\tilde{y} - \tilde{\mathbf{B}}u||^2. \tag{6}$$

For the channel matrix in SD, the number of rows need to be not less than the number of columns. However, the matrix $\tilde{\mathbf{B}}$ is underdetermined $(2F < t)$, and the standard SD can not be applied. It was suggested in [9] to define matrices

$$\bar{y} = \begin{pmatrix} \tilde{y} \\ 0 \end{pmatrix} \in \mathbb{R}^{t+2F}, \bar{\mathbf{B}} = \begin{pmatrix} \tilde{\mathbf{B}} \\ \mathbf{I} \end{pmatrix} \in \mathbb{R}^{(t+2F) \times t},$$

where I is the identity matrix, so that (3) can be reformulated as

$$\hat{u} = \arg\min_{u \in \mathcal{X}} ||\bar{y} - \bar{\mathbf{B}}u||^2.$$

Thus, by using QR factorization one obtains

$$\bar{\mathbf{B}} = \left[\mathbf{Q}_1, \mathbf{Q}_2\right] \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix},$$

where $\mathbf{Q} = \left[\mathbf{Q}_1, \mathbf{Q}_2\right]$ is an $(t + 2F) \times (t + 2F)$ orthogonal matrix and $\mathbf{R}$ is an $t \times t$ upper triangular matrix.

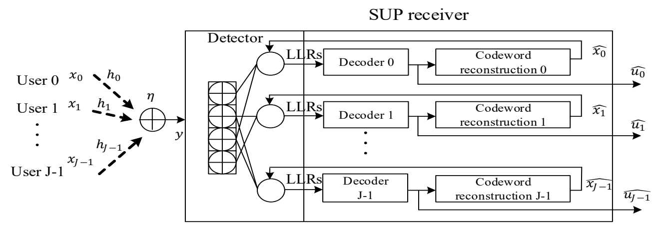

Fig. 2. SUP based PC-NOMA system

The inequality (4) can be written as [5]

$$r^2 \geq \left|\left|\bar{y} - \left[\mathbf{Q}_1, \mathbf{Q}_2\right] \begin{bmatrix}\mathbf{R} \\ \mathbf{0}\end{bmatrix} u\right|\right|^2 = ||\mathbf{Q}_1^T \bar{y} - \mathbf{R}u||^2 + ||\mathbf{Q}_2^T \bar{y}||^2.$$

Since $\mathbf{R}$ is an upper triangular matrix, one can rewrite this vector-valued equations as

$$r^2 \geq \sum_{i=0}^{t-1} \left(\{\mathbf{Q}_1^T \bar{y}\}_i - r_{i,i} u_i - \sum_{j=i+1}^{t-1} r_{i,j} u_j \right)^2, \quad (7)$$

where $r_{i,j}$ denotes (i,j)-element of $\mathbf{R}$. Based on (7), one can detect the elements of u using the successive algorithm. This procedure consists in recursive calculation of upper (UB) and lower (LB) bounds of $\min(\{\mathbf{Q}_1^T \bar{y}\}_l - \sum_{j=l}^{t-1} r_{l,j} u_j)^2$ for each layer l. In [5] lower bounds are calculated before searching, because the lower bound does not depend on value of u_k. When we reach the leaf node of tree, we always change the value of the upper bound for a candidate vector u and reduce the search boundaries. We obtain minimum of UB for all possible values of u as the result.

C. Polar Coding

Polar codes achieve the symmetric capacity of binary-input memoryless channels [10], and have simple encoding and decoding algorithms. The successive cancellation decoding algorithm can be considered as an instance of the multistage decoding algorithm [11], which is commonly used for decoding of multilevel and generalized concatenated codes [12].

Let $W(y|c)$, $c \in \mathcal{C}$, $y \in \mathcal{Y}$ are transition probabilities, where $\mathcal{C}$, $\mathcal{Y}$ are input and output alphabet, respectively. One can define the bit subchannels by the recursion algorithm [10]. Let $\mathcal{F}$ is the set of indices low-capacity subchannels. Then the transmitted symbols of polar code are obtained as $c_0^{n-1} = v_0^{n-1}\mathbf{K}_m$, where $n = 2^m$,

$$\mathbf{K}_m = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m},$$

and v_i, $i \in \mathcal{F}$, are some pre-defined values (frozen symbols).

In [13], [14] application of polar codes in SCMA is studied. There are two types of receiver, which correspond to the SUP (sequential user partition) and PUP (parallel user partition). For the SUP polar-coded scheme, a joint successive cancellation detection and decoding is applied. This approach follows the chain rule of mutual information and resembles channel transform of multilevel coding. For the PUP scheme parallel detection and decoding is used. In this paper we consider only SUP based system (Fig. 2).

One of the best decoding methods of polar codes of length $n = 2^m$ is the Tal-Vardy list decoding algorithm [6]. In this decoder L decoding paths $v_0^{\phi-1}$ are considered concurrently. At each decoder phase at most $2L$ their continuations v_0^{ϕ} are constructed, and L of them with the highest score are selected for further processing. At the end of decoding process, the path with the highest score is returned. We consider the min-sum path score function [15]

$$\begin{aligned} R_m^{(\phi)}(v_0^{\phi}, y_0^{n-1}) &= R_m^{(\phi-1)}(v_0^{\phi-1}, y_0^{n-1}) \\ &+ \tau(S_m^{(\phi)}(v_0^{\phi-1}|y_0^{n-1}), v_{\phi}), \end{aligned} \quad (8)$$

where

$$\tau(S, v) = \begin{cases} 0 & \text{if } \operatorname{sgn}(S) = (-1)^v \\ -|S| & \text{otherwise} \end{cases},$$

and

$$S_{\lambda}^{(2i)}(v_0^{2i-1}|y_0^{2^{\lambda}-1}) = Q(a,b) = \operatorname{sgn}(a)\operatorname{sgn}(b)\min(|a|,|b|),$$

$$S_{\lambda}^{(2i+1)}(v_0^{2i}|y_0^{2^{\lambda}-1}) = (-1)^{v_{2i}} a + b,$$

where

$$a = S_{\lambda-1}^{(i)}(v_{0,e}^{2i-1} \oplus v_{0,o}^{2i-1}|y_0^{2^{\lambda-1}-1}), b = S_{\lambda-1}^{(i)}(v_{0,e}^{2i-1}|y_{2^{\lambda-1}}^{2^{\lambda}-1}).$$

For classical polar codes, a correct path v_0^{n-1} exists with a fairly high probability among L paths obtained by the Tal-Vardy algorithm.

III. Reduced complexity sphere decoder for uncoded SCMA

In this section we propose a reduced-complexity sphere decoding algorithm for SCMA. The proposed approach is based on the one introduced in [16] in the context of MIMO systems.

First of all, we construct an MMSE estimate of vector u not taking into account discrete constraints on it, i.e., we do not take into account that components of this vector belong to set $\mathcal{Q}$. Then, we apply the tree search to find the vector with elements from $\mathcal{Q}$ closest to the obtained estimate. Employing the MMSE estimate enables one to significantly reduce the number of operations performed by the sphere decoder.

Thus, the minimisation problem (6) can be equivalently represented as [16]

$$\hat{u} = \arg\min_{u \in \mathcal{X}} ||\mathbf{D}(\tilde{y} - \hat{s})||^2, \quad (9)$$

where $\mathbf{D}$ is an upper-triangular matrix having positive real-valued elements on the main diagonal and satisfying

$$\mathbf{D}^H \mathbf{D} = (\tilde{\mathbf{B}}^H \tilde{\mathbf{B}} + \sigma^2 \mathbf{I}),$$

while

$$\hat{s} = (\tilde{\mathbf{B}}^H \tilde{\mathbf{B}} + \sigma^2 \mathbf{I})^{-1} \tilde{\mathbf{B}}^H \tilde{y}.$$

Here $\hat{s}$ is an MMSE estimate of the transmitted signal vector u. Note that $\mathbf{D}$ may be obtained by the Cholesky decomposition of the matrix $(\tilde{\mathbf{B}}^H\tilde{\mathbf{B}} + \sigma^2\mathbf{I})$. In the case of AWGN channel, all columns of B have the same norm, but in general one can rearrange them to improve the detection speed.

Let $J(\check{u}) = ||\mathbf{D}(\check{u} - \hat{s})||^2$ be a cost function, which is given by the Euclidean distance between the received signal vector $\tilde{y}$ and a candidate transmitted vector $\check{u}$. We need to find vector $\hat{u}$, such that $\hat{u} = \arg\min_{\check{u}\in\mathcal{X}} J(\check{u})$. Let $\mathcal{J} = J(\hat{u}) = \min_{\check{u}\in\mathcal{X}} J(\check{u})$.

Let the sub-cost functions be defined as

$$\phi_i(\check{u}_i) = \left|\sum_{j=i}^{t-1} d_{ij}(\check{u}_j - \hat{s}_j)\right|^2 = |d_{ii}(\check{u}_i - \hat{s}_i) + a_i|^2,$$

where $a_i = \sum_{j=i+1}^{t-1} d_{ij}(\check{u}_j - \hat{s}_j)$. These function represents the current cost for all users j, such that $j > i$. Thus, we have

$$J(\check{u}) = ||\mathbf{D}(\check{u} - \hat{s})||^2 = \sum_{i=0}^{t-1} \phi_i(\check{u}_i).$$

A reduced complexity sphere decoder for uncoded SCMA system is presented in Algorithm 1. It operates as follows:

Step 1: Initialization

1) Line 1: Construct a real-valued problem given in (5).
2) Line 2 -3: Apply the Cholesky decomposition to obtain an upper-triangular matrix $\mathbf{D}$.
3) Line 4: Calculate the MMSE estimate of transmitted vector $\hat{s}$.
4) Line 5: The record cost $\mathcal{J}$ is set to infinity and $J_t(\check{u}_{i+1})$ is set to 0.
5) Line 6: Run the recursive function *CalculateJ* with argument t to perform the tree search in the descending order from the last layer $t-1$ down to the first layer 0.

Step 2: Calculation of $\mathcal{J}$ and $\hat{u}$

1) Line 2: Calculate a_i, i.e. the contribution of processed symbols $\check{u}_j, j > i$.
2) Line 3: Sort the candidate values q_m in the increasing order of the corresponding values of the sub-cost function $\phi_i(q_m)$.
3) Line 4-15: Enumeration of possible values of q_m.
   - Lines 5-6: Calculate the cost $J_i(\check{u}_i)$ for the candidate $\check{u}_i = q_m$.
   - Line 7: Check if the obtained candidate vector is not worse than the current record.
   - Line 8-9: If we have not reached the first layer, proceed recursively to the previous layer.
   - Line 11-12: Update the radius of the decoding sphere $\mathcal{J}$ and save the obtained $\hat{u}$ as the current record.

IV. Decoding of polar-coded SCMA

In this section we propose a joint decoding algorithm for polar-coded SCMA. To obtain this algorithm, we need first to derive a simple method for soft-output SCMA detection, which provides the input values for the Tal-Vardy successive cancellation list decoder.

Algorithm 1 Reduced Complexity Sphere Decoder

Input: $y \in \mathbb{R}^{F\times 1}$, $\mathbf{B} \in \mathbb{R}^{F\times t}$ and σ^2
Output: $\mathcal{J}$ and $\hat{u}$

1: $\tilde{\mathbf{B}} = \begin{pmatrix}\mathcal{R}(\mathbf{B})\\ \mathcal{I}(\mathbf{B})\end{pmatrix}$, $\tilde{y} = \begin{pmatrix}\mathcal{R}(y)\\ \mathcal{I}(y)\end{pmatrix}$;
2: $\mathbf{R} = (\tilde{\mathbf{B}}^H\tilde{\mathbf{B}} + \sigma^2\mathbf{I})$;
3: $\mathbf{D} =$ CholeskyDecomposition$(\mathbf{R})$;
4: $\hat{s} = \mathbf{R}^{-1}\tilde{\mathbf{B}}^H\tilde{y}$;
5: $\mathcal{J} = \infty,\ J_t(\check{u}_t) = 0$;
6: $CalculateJ(t-1)$;

1: **function** CalculateJ(i)
2: $\quad a_i = \sum_{j=i+1}^{L-1} d_{ij}(\check{u}_j - \hat{s}_j)$;
3: $\quad$ Sort $\{q_m\}$, such that $\phi_i(q_0) < \phi_i(q_1)$, $q_m \in \mathcal{Q}$ where $\phi_i(q_m) = |d_{ii}(q_m - \hat{s}_i) + a_i|^2$;
4: $\quad$ **for** $m = 0, 1$ **do**
5: $\quad\quad \check{u}_i = q_m$;
6: $\quad\quad J_i(\check{u}_i) = J_{i+1}(\check{u}_{i+1}) + \phi_i(\check{u}_i)$;
7: $\quad\quad$ **if** $J_i(\check{u}_i) < \mathcal{J}$ **then**
8: $\quad\quad\quad$ **if** $i > 0$ **then**
9: $\quad\quad\quad\quad CalculateJ(i-1)$;
10: $\quad\quad\quad$ **else**
11: $\quad\quad\quad\quad \mathcal{J} = J(\check{u})$;
12: $\quad\quad\quad\quad \hat{u} = \check{u}$;
13: $\quad\quad\quad$ **end if**
14: $\quad\quad$ **end if**
15: $\quad$ **end for**
16: **end function**

A. Low-complexity soft-output detection

Let us consider an SCMA system which employs some kind of channel coding for each user, and multistage decoding at the receiver. That is, an SCMA code sequence of user j is recovered after the SCMA code sequences of users $0, \ldots, j-1$. Let vector $\mathbf{c}_j^{(i)} = (c_{0,i}, \ldots, c_{dj-1,i})$, $0 \le i < n$ corresponds to already decoded bits for users which indexes are less than j. For simplicity the index i will be omitted. Implementing such kind of receiver requires one to be able to compute log-likelihood ratios

$$\lambda_{j,k}(\mathbf{c}_j, y) = \log\frac{P\left\{\mathbf{c}_{j-1}, c_{dj+k} = 0|y\right\}}{P\left\{\mathbf{c}_{j-1}, c_{dj+k} = 1|y\right\}}, \qquad (10)$$

where $0 \le k < d$. Notice that each c_{dj+k} corresponds to u_{dj+k}, i. e $u_{dj+k} = \mathcal{U}(c_{dj+k})$ (see *Mapping* in Fig. 1), where

$$\mathcal{U}(a) = \begin{cases}1 & a = 1\\ -1 & a = 0\end{cases}.$$

Therefore, we can reformulate (10) as follows

$$\begin{aligned}\lambda_{j,k}(u_0^{dj-1}, y) &= \log\frac{P\left\{u_0^{dj-1}, u_{dj+k} = 1|y\right\}}{P\left\{u_0^{dj-1}, u_{dj+k} = -1|y\right\}}\\ &= \log\frac{\sum_{u\in\mathcal{X}_k^{(1)}(u_0^{dj-1})} P(y|u)}{\sum_{u\in\mathcal{X}_k^{(-1)}(u_0^{dj-1})} P(y|u)}, \qquad (11)\end{aligned}$$

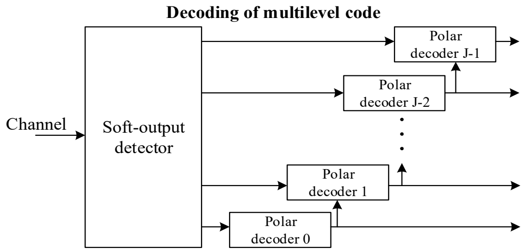

Fig. 3. Classical decoding scheme

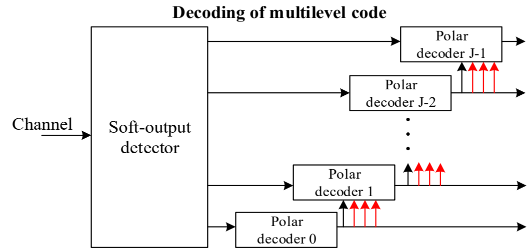

Fig. 4. Joint list decoding scheme

where $P\left\{u_0^{dj-1}, u_{dj+k}=a|y\right\}$ is the probability of transmission of given symbols u_0^{dj-1}, u_{dj+k} given the received noisy vector y, $\mathcal{X}_k^{(e)}(u_0^{dj-1}), 0 \le k < d, e \in \mathcal{Q}$, is a subset of $\mathcal{Q}^t$, which consists of vectors u having the k-th symbol of user j (i.e. u_{dj+k}) equal to e, and symbols $u_i, 0 \le i < dj$, corresponding to users $0, \ldots, j-1$. The second equation of (11) follows from the assumption of uniform distribution of the elements of vector u_0^{t-1}.

We propose to approximate the sums in (11) with the maximal terms. In the case of AWGN channel, this results in

$$\lambda_{j,k}(u_0^{dj-1}, y) \approx \frac{1}{2\sigma^2}\left(F_{\mathcal{X}_k^{(1)}(u_0^{dj-1})} - F_{\mathcal{X}_k^{(-1)}(u_0^{dj-1})}\right), \quad (12)$$

where $F_X = \min_{u \in X} ||y - \mathbf{B}u||^2$ for some set X (see above for the definition of $\mathcal{X}_k^{(e)}(u_0^{dj-1})$). This value can be further represented as

$$F_{\mathcal{X}_k^{(e)}(u_0^{dj-1})} = \min_{\substack{(u_{dj},\ldots,u_{d(j+1)-1}) \in \mathcal{Q}^d \\ u_{dj+k}=e}} F_{X(u_0^{d(j+1)-1})},$$

where $X(u_0^{d(j+1)-1})$ is a set of vectors $u' \in \mathcal{Q}^t$ with $u'_i = u_i, 0 \le i < d(j+1)$.

Hence, (12) can be computed by invoking the above described sphere decoding algorithm (Algorithm 1) M times, which can compute efficiently the values $F_{X(u_0^{d(j+1)-1})}$. Furthermore, since symbols $u_i, 0 \le i < (d+1)j$ are known, their contribution to $\mathbf{B}u$ and y can be subtracted. Let $\mathbf{B}^{(j)}$ be a submatrix of $\mathbf{B}$ consisting of columns $dj, \ldots, t-1$, which correspond to not-yet-processed users. The values $F_{X(u_0^{d(j+1)-1})}$ can be computed by invoking Algorithm 1 with input values $y' = y - \sum_{i=0}^{d(j+1)-1} \mathbf{B}_i u_i$, $\mathbf{B}^{(j)}$, and σ^2.

B. Joint List Polar Decoding

In this section we propose an improved decoding method for polar coded SCMA systems.

The SUP based receiver is very similar to the multistage decoding algorithm for multilevel codes. The successive cancellation decoding algorithm for polar codes can be also considered as an instance of multistage decoding [12]. Hence, polar codes can be naturally combined with the SUP receiver for SCMA. Furthermore, the improved decoding techniques developed in the area of polar coding, such as list decoding [6], can be extended to the case of joint polar decoding and SCMA detection.

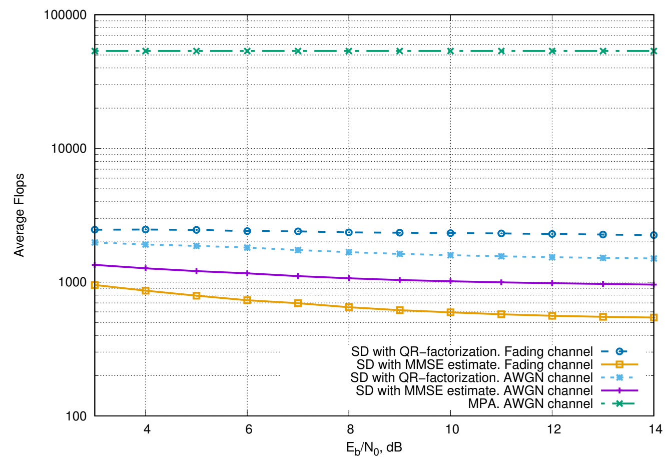

Fig. 5. Average flops of uncoded 4×6 SCMA

That is, instead of selection of a single most probable codeword from the list obtained by the Tal-Vardy decoder for some user, and employing it in the subsequent stages of the multistage decoding algorithm (see Fig. 3), we propose to extend all L paths obtained by the Tal-Vardy decoder to the subsequent stages, as shown in Fig. 4. Furthermore, the initial value of a score of some path for user j is set to the score (8) for this path obtained by user $j-1$. Observe that the complexity of such joint list decoder is still $O(JLn \log n)$, where n is the length of the polar code, L is the decoder list size and J is the number of users.

Note that the input LLRs for the polar decoder for user j can be computed using reduced-dimension matrix $\mathbf{B}^{(j)}$, as described in Section IV-A.

V. Simulation Results

In this section we present simulation results for the proposed detection scheme over AWGN and independent Rayleigh fading channel. We consider 4×6 SCMA system with $J = 6$, $F = 4$, $d_f = 3$ and $d_j = 2$. We employ the codebooks presented in [8].

In Fig. 5 we report the average number of floating-point arithmetic operations (flops) for MPA, SD with QR factorization described in Section II-B, and the proposed SD implementation in the case of AWGN channel. We observe that the complexity of both SD-based algorithms decreases with SNR. The proposed method requires approximately 35%

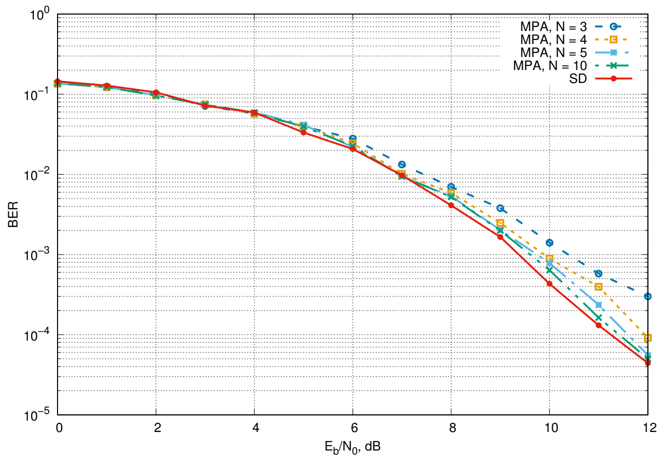

Fig. 6. BER performance of uncoded SCMA over AWGN channel

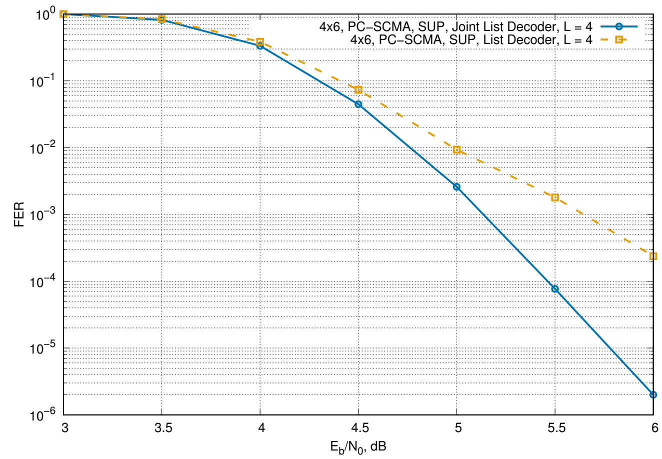

Fig. 7. FER performance of polar coded SCMA over AWGN channel

smaller number of operations at $10\log_{10}(E_b/N_0) = 5$ dB compared to the SD based on QR factorization. Furthermore, the proposed approach provides 50 times complexity reduction with respect to the MPA with 4 iterations. Additionally, in Fig. 5 we report the results for the independent Rayleigh fading channel. It can be seen that the proposed method has approximately 56% lower complexity at $10\log_{10}(E_b/N_0) = 5$ dB compared to the SD based on QR factorization.

Fig. 6 presents the performance of the considered SCMA detection algorithms over AWGN channel. It can be seen that SD outperforms MPA with $N = 10$ iterations. For the case of the Rayleigh fading channel, the performance of SD and MPA with 4 iterations turned out to be identical.

In Fig. 7 we consider FER performance for polar coded SCMA scheme with two decoding methods: classical list decoding and proposed joint list decoding with $L = 4$. For all users we employ polar codes $(1024, 512)$ designed for AWGN channel with $10\log_{10}(E_b/N_0) = 2$ dB. Simulation results show that the proposed joint decoding method outperforms the classical SUP scheme and achieves nearly 0.6 dB gain.

VI. Conclusions

In this paper a reduced complexity sphere decoding algorithm for SCMA systems was presented. It was shown that by pre-computing an MMSE estimate of the transmitted vector one can considerably reduce the number of operations performed by the sphere decoder.

Furthermore, it was shown that the performance of polar coded SCMA systems can be improved by employing joint list successive cancellation decoding.

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