JOINT ADAPTATION OF CODE LENGTH AND MODULATION FORMATS IN OFDM SYSTEMS

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ABSTRACT

In order to reduce the signaling overhead in adaptive OFDM transmission systems being designed for B3G and 4G wireless communications, the basic resource unit, referred to as chunk, comprises a number of adjacent sub-carriers and OFDM symbol periods. A consequent open issue is the mapping of long codewords onto limited size chunks, especially when link adaptation per chunk is carried out. This paper proposes a new method for joint coding and modulation. A data packet, independently of its length, is encoded without any segmentation into a single codeword spanning different chunks employing different signal constellations. The parameters of the underlying error correction code are chosen independently of the signal constellation and no puncturing is required. Moreover, a joint adaptation of the code length and the modulation format is proposed to optimise the number of chunks allocated for transmission.

I. INTRODUCTION

It is widely recognized that link adaptation is highly beneficial to fully exploit the capacity of wideband and broadband frequency selective wireless channels. A great number of bit and power loading (bitloading) algorithms have been proposed during last few years especially for OFDM (Orthogonal Frequency Division Multiplexing) systems [1, 2]. Most of these algorithms are based on the well-known waterfilling principle [3]. Channel coding then is required to guarantee sufficient error protection and meet the QoS demands envisaged for beyond 3G and 4G communications. The easiest way to achieve high spectral efficiency, which is also a requirement in these systems, is that of applying coded modulation techniques. This implies that adaptive coded modulation is needed.

Adaptive transmission, in the form of link adaptation as well as adaptive resource allocation, requires however signaling overhead to acquire a sufficiently accurate channel state information and to schedule resources and transmission formats. In order to reduce such signaling overhead, in different frameworks, e.g. the 3GPP LTE Study Item and the IST Project WINNER [4], [5], it is recommended that the minimum resource unit is comprised of a number of time-frequency bins constituting a rectangular area usually referred to as chunk. The chunk size should be chosen according to the channel correlation properties, e.g. in such a way that within one chunk approximately flat fading and approximately time invariant channel can be observed, and a unique coding/modulation format can be used.

This approach gives rise to a series of new open issues. One of these issues is mapping of long codewords onto limited size chunks. This problem becomes even more difficult when link adaptation is used. In [6], the authors presented a method for the mapping of codewords on chunks based on the concatenation of an outer flexible code with an inner modulation code, that performs joint adaptive coding and modulation per chunk. In this paper, we propose a more general approach for the mapping of data bits, coded or directly uncoded, on different chunks. This allows high granularity in the link adaptation per chunk, while maintaining fully decoupled the choice of the code length from the chunk size. A variation of the semi-BICM coded modulation scheme presented in [7] is here considered for the implementation of the proposed approach and its performance is illustrated.

The remainder of the paper is organized as follows. Section II. describes the semi-BICM scheme and illustrates how it can be applied for chunk-wise adaptive transmission. Section III. considers an OFDMA/TDMA multiple access scheme with chunk-wise adaptive resource allocation. A joint adaptation of code length and modulation format in the coded modulation scheme is proposed towards the optimization of the number of chunks spanned by a codeword. Section IV. presents the numerical results. Finally, some conclusions are drawn.

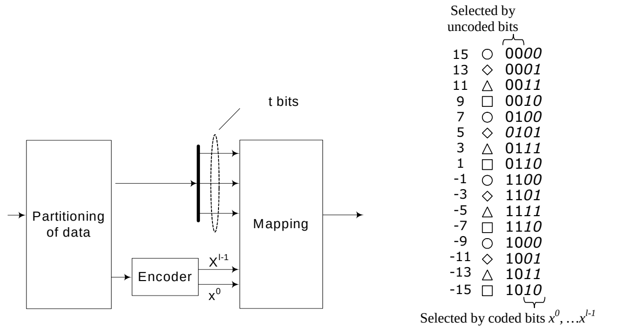
II. PROPOSED CODING METHOD

A. Semi-BICM

Many different coded modulation techniques have been proposed during last 30 years. The most remarkable one is multilevel coding (MLC) [8, 9], which is known to achieve the channel capacity, provided that capacity-approaching component codes are used. Bit interleaved coded modulation (BICM) [10] represents a pragmatic alternative to MLC, which is capable to closely approach its performance, at least for long codes.

The disadvantage of multi-level codes is that they require usage of component codes with length exactly equal to the length of the multi-level codeword. In the context of chunk-based systems, this prevents one from using good long codes. The natural solution is to combine the concepts of BICM and multilevel coding. Namely, the original signal constellation is partitioned into 2^l subsets, where l is some integer. For example, Figure 1(b) illustrates the partitioning of the 16-AM constellation into four subsets, denoted by circles, diamonds, triangles and squares. The subsets are indexed by Gray-encoded labels. Within each subset either Gray or Ungerboeck labeling may be used.

Encoding is performed as follows. The data stream is partitioned into two substreams, as it is illustrated in Figure 1(a).



(a) Encoder

(b) Douple-Gray constellation labeling

Figure 1: Semi-BICM

The first substream is encoded with a (n, k) code. The obtained codewords are partitioned into l-tuples used to select a constellation subset for each of the n/l symbols. The second substream can be kept uncoded. Its bits are grouped into t-tuples identifying a particular point within this subset, for each of the n/l symbols. This approach is denoted here as semi-BICM due to it similarity with the one presented in [7]. Observe, that the obtained Euclidean-space code can be still considered as a multi-level code with just two levels of coding. The first level is protected with a length-n/l code over $GF(2)^l$, and a trivial $(n/l, n/l)$ code over $GF(2)^t$ is used on the second level. Moreover, the proposed method has also some similarity with pragmatic trellis-coded modulation [11].

The advantage of semi-BICM with respect to multi-level coding is that it allows one to employ l times longer binary codes, with potentially better performance. Furthermore, only one coding scheme is employed, while MLC requires in many cases both low-rate and high-rate codes. If, e.g., LDPC (Low Density Parity Check) component codes are used [12], it may be difficult to fine tune their rate as required. The advantage of semi-BICM with respect to BICM is that it decouples the length of the underlying binary code from the size of the signal constellation being used. Indeed, one can use any 2^{l+t}-AM constellation, where $t \geq 0$. Conventional BICM requires implementation of codes with different parameters for each modulation format being used.

The main factor affecting the performance of semi-BICM is the Euclidean distance d between adjacent constellation points. Since the average power of $2^{2(l+t)}$-QAM symbols is given by $\frac{d^2}{6}(2^{2(l+t)} - 1)$, one can assume that the SNR needed to achieve some target codeword error rate is approximately given by $\xi = \Gamma(2^{2(l+t)} - 1)$, where Γ is a factor which depends on the underlying binary code. This is similar to the well-known gap approximation [13].

B. *Mapping of Codewords on Chunks*

Semi-BICM allows one to implement joint coding of chunks with different modulation formats. Indeed, different symbols within a semi-BICM codeword can be taken from different signal constellations, by simply changing the number of uncoded bits t transmitted within one symbol. Depending on the required performance, a fixed and equal number of coded bits l is mapped on each symbol of the allocated chunks, as shown in Figure 2, independently of the signal constellation supported by that chunk. The number of chunks m being coded jointly and, hence, the length of the code obtained, is chosen based on a tradeoff between the required level of error protection, and the number m of chunks that the coded packet can be allocated for transmission.

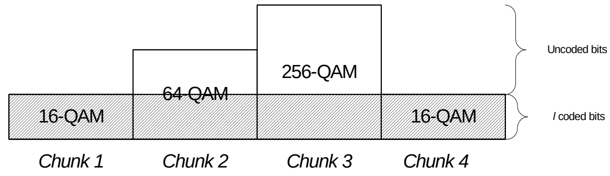


Figure 2: Non-uniform chunk modulation

Let us assume that each chunk consists of $2S\ 2^{l+t_i}$-AM symbols or, equivalently, $S\ 2^{2(l+t_i)}$-QAM symbols, and all symbols in the chunk carry $t_i, i = 1..m$ uncoded bits per dimension, where m is the total number of chunks being encoded jointly. Link adaptation per chunk can be accomplished by varying t_i, $i = 1, 2, ..m$.

Furthermore, assume that a $(2Sml, mk)$ binary component code is used. Then the total number of information bits carried by such semi-BICM codeword is given by

$$K = mk + 2S \sum_{i=0}^{m-1} t_i. \qquad (1)$$

It can be observed that the function $\Gamma(m)$ yielding the required SNR gap in the case of $(2Sml, mk)$ component code is a decreasing function of m, the number of chunks being encoded jointly. Indeed, performing joint coding of a number of chunks allows one to employ longer error-correcting codes, improving thus the performance.

Link adaptation per chunk can be accomplished by varying t_i, $i = 1, 2, ..m$, the number of uncoded bits transmitted within one $2^{2(l+t_i)}$-QAM symbol dimension in the i-th chunk.

III. Joint Optimization of Code Length and Modulation Formats

The main difficulty arising in the design of adaptive coded modulation in an adaptive chunk based OFDM system is that the number of chunks actually used and their modulation formats are random values, which are not known at the design time. According to the classical waterfilling theorem [3], in a frequency-selective channel data should be transmitted only over the best sub-carriers. The use of other sub-carriers could require too high transmit power in order to achieve the required SNR at the receiver. This effectively limits the number of chunks which can be used for data transmission, i.e. the length of the code. These constraints contradict the desire to use long capacity-approaching component codes, e.g. LDPC, in order to achieve strong error protection of data bits. It is therefore necessary to find a tradeoff between good long codes, which need a lot of chunks, and short codes requiring only a few best chunks.

In what follows the joint optimization of code length and modulation format is addressed, in order to optimize the number of allocated chunks m for codeword transmission in terms of minimization of required transmit power.

Let us consider a chunk-based OFDM system, where each chunk consists of L_t OFDM symbols and L_f adjacent subcarriers, i.e. $S = L_t L_f$. In order to fulfill the envisaged stringent requirements on air interface delay, semi-BICM codewords have to be transmitted within one chunk duration, but can span more than one chunk in the frequency direction. The received signals r_i in an OFDM system can be written as

$$r_i = \mu_i V_{\lfloor i/L_f \rfloor} s_i + \eta_i, \eta_i \sim N(0, \sigma^2), i = 0..N-1,$$

where s_i is the symbol transmitted over the i-th subcarrier, selected from $2^{2(l+t_{\lfloor i/L_f \rfloor})}$-QAM constellation, μ_i is the channel transfer factor, V_j is the transmitter gain factor applied to all sub-carriers within chunk j, η_i is an AWGN sample, and $N = ML_f$ is the total number of sub-carriers. The channel is assumed to be time-invariant. It is convenient to introduce the sub-carrier channel-to-noise ratio (CNR) $\chi_i = \frac{|\mu_i|^2}{\sigma^2}$, and the average chunk CNR $\hat{\chi}_j = \left(\prod_{i=jL_f}^{jL_f+L_f-1} \chi_i \right)^{\frac{1}{L_f}}$. Without loss of generality, assume that $\hat{\chi}_0 \geq \hat{\chi}_1 \geq \ldots \geq \hat{\chi}_{M-1}$. Data transmission has to be performed over a number $m \leq M$ of chunks with highest CNR $\hat{\chi}_j$. For each active chunk j the transmitter has to compute the number of uncoded bits per symbol $t_j, j = 0..m-1$, and the power gain V_j, so that the average chunk signal-to-noise ratio at the receiver satisfies

$$\hat{\chi}_j V_j^2 = \Gamma(m)(2^{2(l+t_j)} - 1), j = 0..m-1.$$

Observe that it may be necessary to introduce the per-subcarrier gain factors $V_{jl}, l = 0..L_f - 1$, in order to compensate frequency selectivity of the channel.

In this scenario, it is not a-priori clear if it is better to employ a short semi-BICM code over just a few chunks with the highest CNR, or activate some not so good chunks, by employing longer and better code with smaller $\Gamma(m)$. Hence, the optimization problem can be formulated as minimization of the total transmit power

$$\min_{m,t_j} P(m, t_0, \ldots, t_{m-1}),$$

where

$$\begin{aligned} P(m, t_0, \ldots, t_{m-1}) &= L_t L_f \sum_{j=0}^{m-1} V_j^2 \qquad (2) \\ &= L_t L_f \Gamma(m) \sum_{j=0}^{m-1} \frac{2^{2(l+t_j)} - 1}{\hat{\chi}_j}, \end{aligned}$$

with total data rate constraint

$$K = mk + 2L_t L_f \sum_{j=0}^{m-1} t_j,$$

where k is the dimension of the code to be used for single-chunk coding.

The described optimization problem belongs to the class of integer programming problems. Its exact solution may require considerable computational effort. The complexity can be reduced by allowing t_j to take continuous values. For a fixed m, the Lagrangian of this problem is given by

$$\begin{aligned} \mathcal{L}(t_i, \lambda) &= L_t L_f \Gamma(m) \sum_{j=0}^{m-1} \frac{2^{2(l+t_j)} - 1}{\hat{\chi}_j} \\ &- \lambda \left(mk + 2L_t L_f \sum_{j=0}^{m-1} t_j - K \right). \end{aligned}$$

This leads to the following system of equations:

$$\begin{aligned} \Gamma(m) \ln 2 \frac{2^{2(l+t_j)}}{\hat{\chi}_j} &= \lambda \\ 2L_t L_f \sum_{j=0}^{m-1} t_j &= K - mk \end{aligned}$$

Letting $2t_j = \log_2\left(\frac{\lambda \hat{\chi}_j}{\Gamma(m) \ln 2}\right) - 2l$, one obtains

$$\log_2\left(\frac{\lambda}{\Gamma(m) \ln 2}\right) = 2l - \frac{1}{m} \sum_{j=0}^{m-1} \log_2(\hat{\chi}_j) + \frac{K - mk}{mL_t L_f},$$

or

$$2t_i = \log_2(\hat{\chi}_i) - \frac{1}{m} \sum_{j=0}^{m-1} \log_2(\hat{\chi}_j) + \frac{K - mk}{mL_t L_f}, i = 0..m-1. \qquad (3)$$

The values t_i obtained from this expression may be non-integer or even negative and must be appropriately rounded, making thus the solution suboptimal. Observe that (3) depends on m, but not on $\Gamma(m)$. However, (2) does depend on it, and has to be optimized over m. In principle, one can consider all possible values of m and find the required average power using the expression in (2). If $\Gamma(m)$, considered as a function of a real-valued argument, is convex, the complexity can be reduced by employing the golden section optimization method [14].

Hence, the optimization algorithm involves the following steps:

1. Order chunks according to their average CNR.
2. Use (3) to compute t_i, round it appropriately, and substitute it into (2) in order to obtain total transmit power $P(m)$ for $m = m_l = 1$ and $m = m_h = M$.
3. Let $m_1 = \lfloor 0.382m_h + 0.618m_l \rfloor$ and $m_2 = \lceil 0.382m_l + 0.618m_h \rceil$ Perform the same calculations for $m = m_1$ and $m = m_2$. Let $m_h = m_2$ if $P(m_1) < P(m_2)$, and $m_l = m_1$ otherwise. Repeat this step until $m_h - m_l < 4$.

4. Let $m_0 : m_l \leq m_0 \leq m_h$ be the number of active chunks minimizing the total transmit power. Return $((m_0, (t_0, \ldots, t_{m_0-1}))$.

The algorithm returns the number of chunks being coded jointly, and the proper modulation format for each chunk.

IV. Numeric results

Figure 3 illustrates the performance of semi-BICM with $l = 2$ coded bits per dimension in AWGN channel. $(384m, 192m), m = 1, 2, \ldots$ interleaver-permutation LDPC codes [15] were used as component codes. For comparison, the curves corresponding to pragmatic TCM [11] with the same block length are also presented. As it may be expected, long semi-BICM codes provide better performance than shorter ones, and considerably outperform pragmatic TCM. However, the use of only one level of coding leads to a slowdown of the curves corresponding to long codes at $FER = 3 \cdot 10^{-3}$ in the case of $t > 0$ (Figure 3(b)). This is caused by errors occuring in the uncoded bits of the signal constellation. If this behaviour is not acceptable, e.g. if ARQ is not used, one may need to replace uncoded bits (see Figure 1(a)) with the output of some high rate algebraic encoder. However, this would require some modifications to the derivations presented in Section III.

Figure 4 presents simulation results for an adaptive single-user OFDM system using the proposed mapping of semi-BICM codewords onto chunks and the above described optimization algorithm for joint adaptation of code length and modulation. The system has 512 sub-carriers, 20 MHz bandwidth, and is assumed to operate in an Urban Micro channel modelled as in [16]. The channel is assumed to be time-invariant for the duration of L_t OFDM symbols. The results are reported in terms of transmit power required to achieve a given throughput per OFDM symbol, which is defined as $T = \frac{K(1-FER)}{L_t}$. Link adaptation was performed on the per-chunk basis. The cases of $L_t = 1, L_f = 96$ and $L_t = 8, L_f = 12$ are considered. For comparison, the performance of conventional bit and power loading [1] combined with independent rate 2/3 LDPC coding (i.e. adaptive BICM) is also shown. It can be seen that if the data rate is low, conventional bit and power loading provides slightly better performance. For higher data rates semi-BICM starts to outperform it in both considered cases. The reason is that in the case of low data rate it is sufficient to employ low-order modulation formats like QPSK on most of the chunks. Such signal constellations just do not have enough space to implement semi-BICM, which needs at least $l = 2$ coded bits per dimension for the considered family of codes. This problem can be avoided by decreasing l. This may, however, require implementing coding at the second level of the generalized multilevel code (see Figure 1(a)). Achieving higher data rates requires employing higher order modulation formats on some chunks. Since different chunks employ different modulation formats, it is not possible to adapt the rate of the binary code used in the BICM system to parameters of all chunks simultaneously, causing it to be suboptimal. On the contrary, semi-BICM can be adapted easily, providing thus higher throughput than pure adaptive BICM.

V. Conclusions

In this paper a novel method for performing adaptive coding and modulation in OFDM systems is presented. The method represents an efficient solution to the problem of mapping codewords on different resource units experiencing different propagation conditions and hence supporting different modulation formats. It has to be observed that the method can be applied also when each time-frequency resource unit is reused in the spatial or code domain. Its investigation jointly with multiple antenna techniques, e.g. SDMA (space division multiple access), is subject of further study.

The coding approach is based on semi-BICM coded modulation scheme, which can be considered as a combination of well-known BICM and MLC techniques. This enables one to encode an entire data packet, independently of its length, into a single codeword spanning different chunks, using chunk-specific modulation formats. Moreover, an optimization algorithm is proposed to jointly adapt the modulation format and the code-length to optimize the number of spanned chunks. The proposed approach outperforms conventional BICM combined with bit and power loading in the high data rate region, where high-order modulation formats have to be used on some chunks.

Acknowledgement

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References

[1] B. Krongold, K. Ramchandran, and D. Jones, "Computationally efficient optimal power allocation algorithms for multicarrier communication systems," *IEEE Transactions on Communications*, vol. 48, no. 1, pp. 23–27, January 2000.

[2] N. Papandreou and T. Antonakopoulos, "A new computationally efficient discrete bit-loading algorithm for DMT applications," *IEEE Transactions On Communications*, vol. 53, no. 5, May 2005.

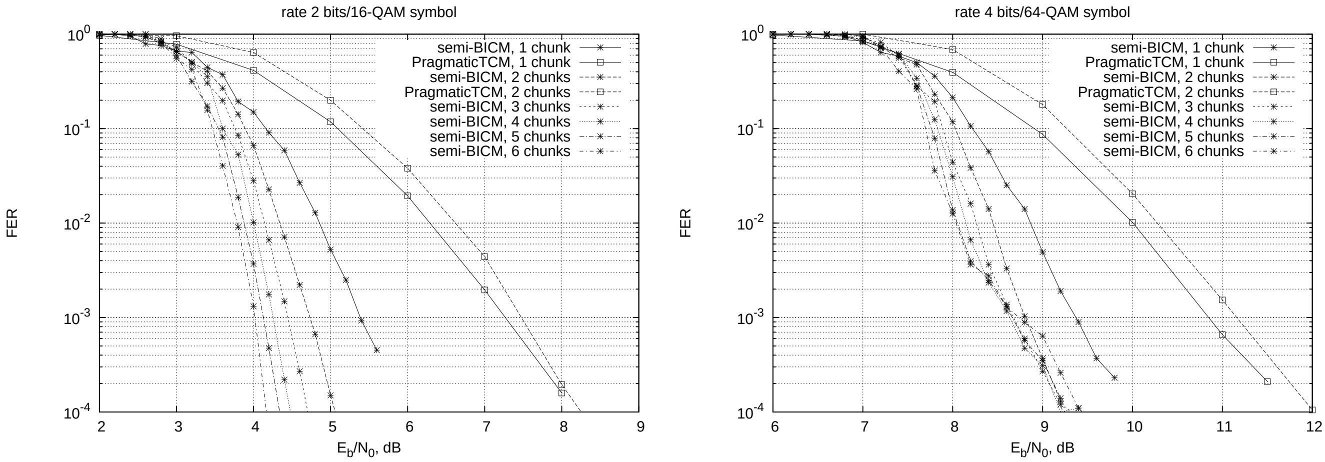
[3] R. G. Gallager, *Information Theory and Reliable Communication*. New York: Wiley, 1968.

[4] "R1-051365: Frequency bandwidth of resource blocks for DL OFDMA," 3GPP, Tech. Rep., 2005.

[5] "D2.10: Final report on identified key radio interface technologies, system concepts and their assessment," IST-2003-507581 WINNER, Tech. Rep., 2005.

[6] E. Costa, P. Trifonov, and E. Schulz, "Method for mapping data packets onto physical layer resource units in an adaptive transmission system," in *Proceedings of 14th Wireless World Research Forum*, San-Diego, July 2005.

[7] J. Hou and M. H. Lee, "Multilevel LDPC codes design for semi-BICM," *IEEE Communications Letters*, vol. 8, no. 11, November 2004.



(a) No uncoded bits

(b) $t = 1$ uncoded bit per AM symbol

Figure 3: Comparison of semi-BICM and pragmatic TCM

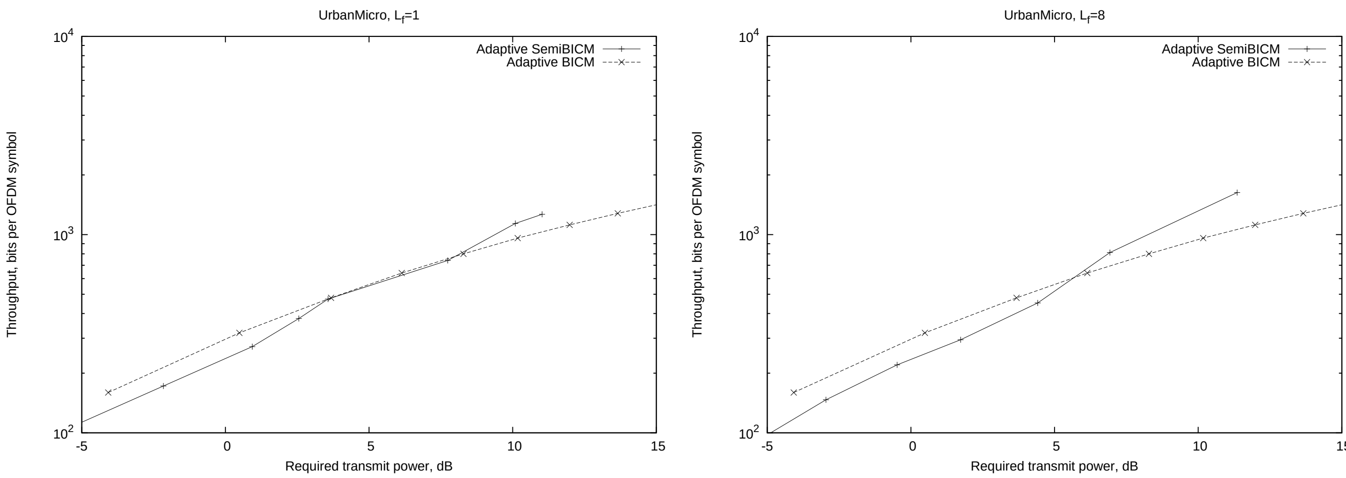


Figure 4: Performance of adaptive semi-BICM system

[8] H. Imai and S. Hirakawa, "A new multilevel coding method using error correcting codes," *IEEE Transactions on Information Theory*, vol. 23, no. 3, pp. 371–377, May 1977.

[9] U. Wachsmann, R. F. H. Fischer, and J. B. Huber, "Multilevel codes: Theoretical concepts and practical design rules," *IEEE Transactions On Information Theory*, vol. 45, no. 5, pp. 1361–1391, July 1999.

[10] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Transactions on Information Theory*, vol. 44, pp. 927–946, May 1998.

[11] A. Viterbi, J. K. Wolf, E. Zehavi, and R. Padovani, "A pragmatic approach to trellis-coded modulation," *IEEE Communications Magazine*, vol. 27, no. 7, pp. 11–19, 1989.

[12] W. E. Ryan, "An introduction to LDPC codes," in *CRC Handbook for Coding and Signal Processing for Recording Systems*, B. Vasic, Ed. CRC Press, 2004.

[13] G. D. Forney and M. V. Eyuboglu, "Combined equalization and coding using precoding," *IEEE Communications Magazine*, vol. 29, no. 12, Dec. 1991.

[14] N. N. Kalitkin, *Numerical methods*. Moscow: Nauka, 1978, in Russian.

[15] P. Trifonov and E. Costa, "Construction of structured irregular LDPC codes," *IEEE Communications Letters, submitted for publication*, 2006.

[16] D. S. Baum, J. Hansen, G. D. Galdo, M. Milojevic, J. Salo, and P. Kyösti, "An interim channel model for beyond-3G systems: Extending the 3GPP spatial channel model (SCM)," in *Proceedings of VTC'05-Spring*, 2005.