Adaptive User allocation, Bit and Power Loading in Multi-Carrier Systems

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***Abstract*—Multi-user sub-carrier allocation, bit and power loading for OFDM-based wireless systems are considered. We show that a proper optimization of the sub-carrier sharing, e.g. by means of CDMA, enables a considerable performance gain. The gain increases noticeably with the number of users.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is considered as the most promising solution for high data rate communications in frequency-selective multi-path channels. The main reason for this is the relatively simple receiver architecture [3]. However, some kind of link adaptation is required in order to obtain good performance. The single-user waterfilling theorem states that the transmitter should avoid using the sub-carriers with poor channel transfer factors [**?**]. For other sub-carriers, transmitter gains and modulation formats should be adjusted in order to achieve some target quality of service. A great number of algorithms have been suggested for this problem for the single-user case, cf. [1], [5], [6], [7], [8], [9], [10]. Typically, these algorithms assume that a fixed bit error rate (BER) is required and either minimize the transmit power with fixed user rate or maximize the user rate with fixed transmit power. These algorithms, referred to as *bit and power loading* (or *bitloading*), can be considered as a practical implementation of the Shannon's waterfilling theorem [11].

However, for the case of multi-user systems the problem of link adaptation is much more difficult. In [4] it is shown that the *total* capacity of a multi-user system can be achieved by frequency-division multiple access (FDMA) if each sub-carrier is assigned to the user having the best channel transfer factor over this sub-carrier. However, this does not ensure any QoS for separate users. It is also shown there that maximization of the weighted sum of user capacities can be achieved by implementing some kind of sub-carrier sharing.

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If one takes into account practical constraints, like discreteness of modulation formats and sub-carrier sharing factors, one obtains a discrete optimization problem which is quite difficult to solve. In [13] a Lagrangian relaxation approach has been used to solve the problem of user to sub-carrier allocation in an OFDM-FDMA (OFDMA) system with fixed user data rate requirements. The exact solution of the relaxed problem includes sub-carrier sharing factors, but, since in OFDMA systems no sharing is possible, the solution is appropriately discretized.

In this paper, we show that it is possible to obtain considerable performance gain by taking sub-carrier sharing into account. We propose an optimization algorithm for the computation of the sharing factors with the aim of minimizing the transmit power while maintaining individual users' data rate requirements. We apply this approach to an OFDM system in which sharing is implemented using CDMA in time domain, i.e. MC-DS-CDMA system [5], and investigate the gain in power efficiency with respect to OFDMA.

The paper is organized as follows. Section II contains the matematical formulation of the problem and the description of the optimization algorithm. Section III presents simulation results illustrating the performance of the algorithm. Finally, some conclusions are drawn in Section IV.

II. Optimization of sub-carrier sharing in a multi-user system

A. Problem statement

Let us consider a multi-carrier downlink system and assume that the signal received by user k over sub-carrier i in one OFDM symbol period can be expressed as

$$r_{ki} = \mu_{ki} s_i + \eta_{ki}, k = 1..K, i = 1..N,$$

where s_i is the signal transmitted by the base station over the i-th sub-carrier and η_{ki} represents Gaussian noise with variance σ^2. Let us assume that the users can share sub-carriers in some way, using e.g. TDMA or CDMA, and let $\rho_{ki} \in [0,1]$ denote the fraction of the i-th sub-carrier occupied by the k-th user. This may be a fraction of time slots for the case of TDMA, or a fraction of spreading sequences for the case of CDMA. Let M_{ki} denote the size of modulation constellation and let V_{ki} be the transmitter gain of user k over sub-carrier i. Observe that the transmitter gain is applied to modulated symbols of individual users before they are superimposed to form the transmitted signals s_i. For the case of TDMA this means that different gains are used for different time slots; for the case of CDMA this means that spreading sequences used by different users are scaled by different factors V_{ki}.

Moreover, let us assume that there is no multiple access interference. This can be easily achieved by TDMA or, if the channel does not change in time, by CDMA with orthogonal spreading sequences, e.g. Walsh-Hadamard ones.

Let us further assume that the data rate required by the k-th user is

$$R_k = \sum_{i=1}^{N} \rho_{ki} c_{ki}$$

bits per one frame, where $c_{ki} = \log_2 M_{ki}$ is the number of bits transmitted by the k-th user over sub-carrier i.

By frame we denote the minimal number of OFDM symbols required to implement the sharing. Here it is assumed that sub-carrier sharing is implemented using CDMA, with S Walsh-Hadamard spreading codes of length S. Then R_k is the number of bits transmitted within a frame consisting of S OFDM symbols, that is optimization is performed over frames. This means that $\rho_{ki} \in \{0, 1/S, \ldots, (S-1)/S, 1\}$. By increasing S the accuracy of the derivations presented below is improved since better approximation of real-valued sharing factors is obtained.

Following [13], let us define $f(c)$ as the function specifying the SNR required to achieve the target BER with data rate c. For the case of 2^c-QAM this function looks like $f(c) = 2\gamma(2^c - 1)/3$, where γ depends on the target BER. Let now $\xi_{ki} = \frac{|\mu_{ki}|^2}{\sigma^2}$ be the channel-to-noise ratio for user k over sub-carrier i. Then the transmitter gain should be equal to $V_{ki} = \sqrt{f(c_{ki})/\xi_{ki}}$.

Now the optimization problem may be stated as

$$\min_{c_{ki},\rho_{ki}} \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{\rho_{ki} f(c_{ki})}{\xi_{ki}} \tag{1}$$

subject to

$$\begin{aligned} R_k &= \sum_{i=1}^{N} \rho_{ki} c_{ki} && (2)\\ 1 &= \sum_{k=1}^{K} \rho_{ki} && (3)\\ 0 &\le \rho_{ki}. && (4) \end{aligned}$$

Through the change of variables $p_{ki} = c_{ki}\rho_{ki}$, as in [13], one obtains the convex programming problem with Lagrangian

$$\begin{aligned} \mathcal{L} &= \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{\rho_{ki}}{\xi_{ki}} f\left(\frac{p_{ki}}{\rho_{ki}}\right) - \sum_{k=1}^{K} \lambda_k \left(\sum_{i=1}^{N} p_{ki} - R_k\right) \\ &- \sum_{i=1}^{N} \beta_i \left(\sum_{k=1}^{K} \rho_{ki} - 1\right) - \sum_{i=1}^{N} \sum_{k=1}^{K} \phi_{ki} \rho_{ki}, \end{aligned}$$

where $\lambda_k, \beta_i, \phi_{ki}$ are Lagrange multipliers. Applying the Kuhn-Tucker theorem [12], after some transformations the following system of equations and inequalities can be obtained

$$\begin{aligned} 0 &= \left(\beta_i^{(k)} - \beta_i\right) \rho_{ki} && (5)\\ R_k &= \textstyle\sum_{i=1}^{N} \rho_{ki} f'^{-1}(\lambda_k \xi_{ki}) && (6)\\ 1 &= \textstyle\sum_{k=1}^{K} \rho_{ki} && (7)\\ \beta_i &\le \frac{f(f'^{-1}(\lambda_k\xi_{ki})) - \lambda_k \xi_{ki} f'^{-1}(\lambda_k \xi_{ki})}{\xi_{ki}} = \beta_i^{(k)}. && (8) \end{aligned}$$

B. *Optimization algorithm*

The main difficulty in solving the system (5)–(8) is caused by the non-uniqueness of the solution. Indeed, let $N = K = 2$ and let us assume that both users see flat fading channels. Then the system performance appears to be independent of the sub-carrier allocation, i.e. many solutions exist. This problem prevents one from applying standard numeric techniques like iterative Newton-type solvers [2].

However, observe from (6) that, for a given user-to-sub-carrier allocation $\{\rho_{ki}\}$, λ_k is uniquely determined by R_k. On the other hand, as it is observed in [13], from (5) it follows that β_i should be equal to

$\min_k \beta_i^{(k)}$ and only users with $\beta_i^{(k)} = \beta_i$ are allowed to use sub-carrier i. This suggests the following optimization approach, which can be considered as a variation of the coordinate descent algorithm:

1) Produce some initial sub-carrier allocation $\{\rho_{ki}\}$. This can be done for example by assigning sub-carriers according to the single-user waterfilling theorem.
2) Compute λ_k from (6) and substitute it into (8) to obtain $\beta_i^{(k)}$.
3) Find the worst allocated sub-carrier, i.e. the one given by $\arg\max_i \max_{k:\rho_{ki}>0} (\beta_i^{(k)} - \beta_i)$.
4) Reduce the fraction of this sub-carrier $\rho_{k_{\text{worst}},i}$ occupied by the user k_{worst} with greatest $\beta_i^{(k)}$ by $1/S$, and increase the fraction $\rho_{k_{\text{best}},i}$ of the sub-carrier assigned to the user k_{best} with the smallest $\beta_i^{(k)}$ by the same value.
5) Repeat steps 2 — 4 until it is not possible to make further changes.

Note that after each iteration of the algorithm, $\beta_i^{(k)}$ values should be updated according to (8) only for two users, $k_{worst} = \arg\max_k \beta_i^{(k)}$ and $k_{best} = \arg\min_k \beta_i^{(k)}$. This greatly simplifies computations.

Observe, that this algorithm automatically takes into account the discreteness of the sub-carrier sharing factors. Moreover, it does not depend on "small values" used in [13] for initialization and updating of λ_k. These values seem to considerably affect both the solution and the complexity. Another important difference from Wong's algorithm is the starting point: in our algorithm it is the initial sub-carrier allocation, which is adjusted in order to satisfy optimality conditions, while in the Wong's algorithm it is the waterfilling constant λ which is adjusted in order to obtain the target rate. This considerably improves the performance.

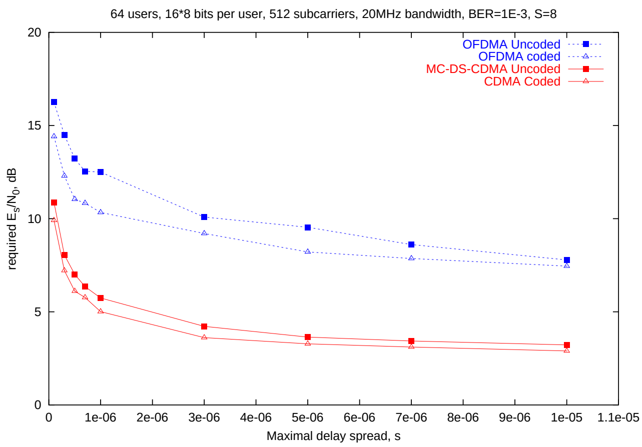
III. Simulation Results

The performance of the suggested algorithm, as well as the one in [13] has been studied by means of simulations. WSSUS channel model with exponential power delay profile has been used. Table I summarizes the system parameters.

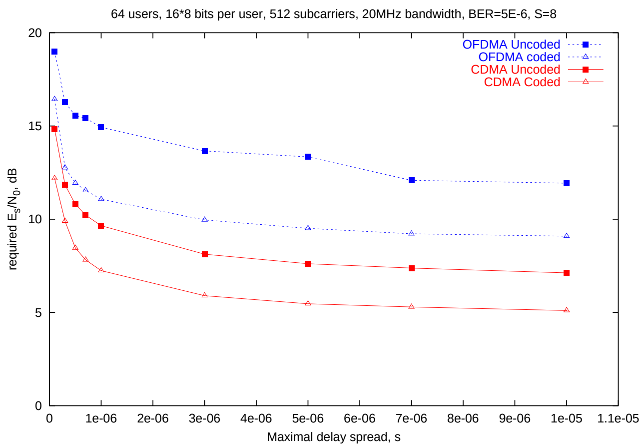
Figures 1 (a) and (b) present simulation results for the case of $K = 64$ users for target BER values equal to 10^{-3} and $5 \cdot 10^{-6}$. Both uncoded and rate $1/2$ convolutionally coded cases are reportted. It can be inferred that by optimizing the sub-carrier sharing by means of CDMA up to 5 dB gain in power efficiency compared to OFDMA system can be obtained. Also,

TABLE I
SIMULATION SETUP

Channel model	WSSUS
Power delay profile	exponential
Bandwidth	20MHz
Number of sub-carriers N	512
Spreading sequence length S	8
Number of bits per user per frame R_k	16*S



(a) Target BER= 10^{-3}



(b) Target BER= $5 \cdot 10^{-6}$

Fig. 1. SNR required to achieve the target BER, $K = 64$

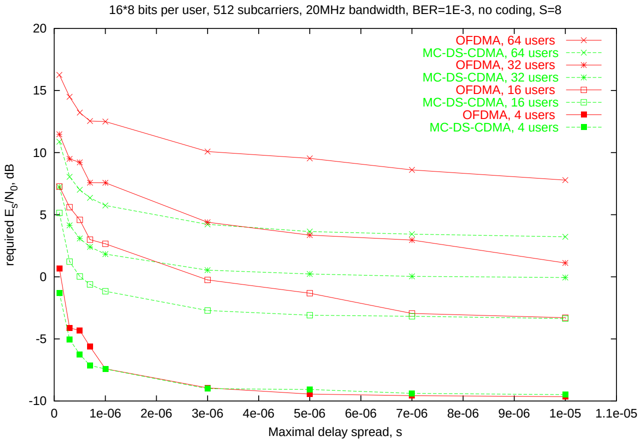


Fig. 2. System performance for different number of users

as it may be expected, coding gain increases as target BER decreases.

Figure 2 presents simulation results for the case of different number of users in the system. One can see that the gain provided by optimized sub-carrier sharing quickly increases with the number of users. This happens because for small number of users it is extremely unlikely that the best sub-carriers coincide for two or more users. Therefore there is no need to perform sub-carrier sharing and the MC-DS-CDMA system reduces to OFDMA.

IV. Conclusions

The performance of a multi-user multi-carrier system depends not only on modulation formats and power distribution of separate users, but also on how users have been assigned to sub-carriers. In this paper we have presented a multi-user multi-carrier system with time-domain spreading and an algorithm for optimization of sub-carrier sharing. If the number of users is sufficiently high, it is very likely that the best sub-carriers for a number of users coincide, that is conflicts occur. In this case these sub-carriers must be shared and the sharing factors have be optimized properly. The proposed algorithm for optimization of the sharing factors provides gain up to 5 dB compared to the optimized OFDMA system.

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